SEC. 13–9/SHEAR AND MOMENT FORMULAS



TABLE 13–1 Shear and Moment Formulas for Some Simple Loadings

473

TABLE 15–2 Allowable Stresses for Timber

| | | | | Compre | ssion |
|-----------------------|--|---|--|--|--------------------------------------|
| Species | Extreme Fiber in Bending psi (kPa) | Tension Parallel to Grain psi (kPa) | Longi- tudinal Shear psi (kPa) | Perpen- dicular to Grain psi (kPa) | Parallel to Grain psi (kPa) |
| Douglas fir | 1450 | 625 | 95 | 385 | 1050 |
| Eastern hemlock | (10 000) 1350 (9310) | (4310) 925 (6380) | (660) 80 (550) | (2650) 360 (2480) | (7240) 950 (6550) |
| Southern pine | 1600 | 825 | 90 | 410 | 1250 |
| Ponderosa pine | (11000) 1100 (7580) | (5690) 725 (5000) | (620) 65 (450) | (2830) 235 (1620) | (8620) 750 (5170) |
| California redwood | 1350 (9310) | 650 (4480) | 100 (690) | 270 (1860) | 1050 (7240) |

| Beam Loading and Deflection | Maximum Deflection | Slope at End(s) | Deflection Equations |
|--|---|---|---|
| $A = \begin{bmatrix} P & \delta_{\max} \\ B & A \\ \hline & & B \\ \hline & & & & \\ \hline & & & & \\ \hline & & & & & \\ \hline & & & &$ | $\delta_{\max} = \frac{PL^3}{3EI}$ | $\theta_B = \frac{PL^2}{2EI}$ | $\delta = \frac{P\mathbf{x}^2}{6EI}(3L - \mathbf{x})$ |
| $\begin{array}{c} 2 & & P \\ A & & C \\ A & & C \\ A & & B \\ A & & $ | $\delta_{\max} = \frac{P\alpha^2}{6EI}(3L - \alpha)$ | $\theta_B = \frac{P\alpha^2}{2EI}$ | $\delta_{AC} = \frac{Px^2}{6EI}(3\alpha - x)$ $\delta_{CB} = \frac{P\alpha^2}{6EI}(3x - \alpha)$ |
| $\begin{array}{c} 3 \\ A \\ \downarrow \\ \downarrow \\ L \\ \downarrow \\ L \\ \downarrow \\ H \\ H$ | $\delta_{ m max} = rac{wL^4}{8EI}$ | $\theta_B = \frac{wL^3}{6EI}$ | $\delta = \frac{\mathbf{w}\mathbf{x}^2}{24EI}(\mathbf{x}^2 - 4L\mathbf{x} + 6L^2)$ |
| $A = \begin{bmatrix} M \\ B \\ \downarrow \\ L \\ \downarrow \\ H \\ H$ | $\delta_{\max} = \frac{ML^2}{2EI}$ | $\theta_{B} = \frac{ML}{EI}$ | $\delta = \frac{Mx^2}{2EI}$ |
| $5 \xrightarrow{L} \xrightarrow{P} C$ | $\delta_{\max} = \frac{PL^3}{48EI}$ | $\theta_{\rm A} = \theta_{\rm B} = \frac{PL^2}{16EI}$ | $\delta_{AC} = \frac{P\mathbf{x}}{48EI}(3L^2 - 4\mathbf{x}^2)$ |
| $\begin{array}{c} 6 \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$ | For $\alpha > b$: $\delta_{\max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL}$ $\alpha t x_m = \sqrt{\frac{L^2 - b^2}{3}}$ | $\theta_{A} = \frac{Pb(L^{2} - b^{2})}{6EIL}$ $\theta_{B} = \frac{P\alpha(L^{2} - \alpha^{2})}{6EIL}$ | $\begin{split} \delta_{AC} &= \frac{Pbx}{6EIL}(L^2 - x^2 - b^2) \\ \delta_{CB} &= \frac{Pb}{6EIL} \bigg[\frac{L}{b} (x - \alpha)^3 \\ &+ (L^2 - b^2) x - x^3 \bigg] \end{split}$ |
| $\begin{array}{c} 7 & w \\ A & 7 & 7 & 7 \\ A & 7 & 7 & 7 \\ 7 & 7 & 7 \\ 7 & 7$ | $\delta_{\max} = \frac{5wL^4}{384EI}$ | $	heta_{A}=	heta_{B}=rac{\mathbf{w}L^{3}}{24EI}$ | $\delta = \frac{\mathbf{w}\mathbf{x}}{24EI}(L^3 + \mathbf{x}^3 - 2L\mathbf{x}^2)$ |
| $\begin{array}{c c} 8 & & & \\ A & $ | $\delta_{\max} = \frac{ML^2}{9\sqrt{3}EI}$ at $\mathbf{x}_m = \frac{L}{\sqrt{3}}$ | $	heta_{A}=rac{ML}{6EI}$ $	heta_{B}=rac{ML}{3EI}$ | $\delta = \frac{M\mathbf{x}}{6EIL}(L^2 - \mathbf{x}^2)$ |

TABLE 16–1 Beam Deflection Formulas

| Type of Load | Type of Stress | Formula | Equation Number |
|---|---|-------------------------------------|--------------------|
| Axial load | Direct normal stress | $\sigma = \frac{P}{A}$ | (9–1) |
| Internal pressure in thin-walled vessels | Circumferential stress | $\sigma_{c} = \frac{Pr_{i}}{t}$ | (9–16) |
| | Longitudinal stress | $\sigma_l = \frac{\Pr_i}{2t}$ | (9–17) |
| Beam bending load | Flexural stress | $\sigma = \frac{My}{I}$ | (14–3) |
| | | $\sigma_{\rm max} = \frac{Mc}{I}$ | (14–2) |
| | | $\sigma_{\rm max} = \frac{M}{S}$ | (14–7) |
| Direct shear load | Direct shear stress | $\tau_{avg} = \frac{P}{A}$ | (9–4) |
| Torque in circular shaft | Torsional shear stress | $\tau = \frac{T\rho}{J}$ | (12-2) |
| | | $\tau_{\rm max} = \frac{Tc}{J}$ | (12–1) |
| Beam shear force | Beam shear stress | $\tau = \frac{VQ}{It}$ | (14–10) |
| | Maximum shear stress in rectangular section | $\tau_{\rm max} = 1.5 \frac{V}{A}$ | (14–11) |
| | Maximum shear stress in circular section | $\tau_{\rm max} = \frac{4V}{3A}$ | (14–12) |

| TABLE 18–1 Lis | t of the | e Fundαment | tal Formulas |
|----------------|----------|-------------|--------------|
|----------------|----------|-------------|--------------|

18–2 COMBINED AXIAL AND BENDING STRESSES

Many structural and machine members are subjected to axial forces and bending moments exerted simultaneously. Both produce normal stresses along the longitudinal directions. The normal stresses due to each load can be calculated separately and added algebraically to find the combined stresses, as illustrated in the following two examples.

- EXAMPLE 18-1 -

Refer to Fig. E18–1(1). The wide-flange shape W360 \times 0.99 is used as a simple beam of 3-m span. The beam is subjected to a uniform load *w* of 100 kN/m (including the weight of the beam) and an axial tensile force *P* of 500 kN. Determine the normal stresses at points *A* and *B*, and plot the normal stress variation between *A* and *B*.